The above considered negative decompensation of pressure perturbations in the first instance along the contour of the body is eliminated by the introduction of a contour discontinuity [3, 8, 9]. This result was obtained in investigations of the properties of the first variation of the minimizing functional. The problem of negative decompensation elimination in the second order can be solved by analyzing the necessary conditions of the Legendre kind.

The author thanks K.A. Lur'e for his assistance and constant interest and also A.N. Kraiko and A.V. Shipilin for their valuable advice.

REFERENCES

1. Shmyglevskii, Iu. D., Variational problems for supersonic bodies of revolution and nozzles. PMM Vol. 26, № 1, 1962.
2. Guderley, K. G., Tabak, D., Breiter, M. C. and Bhutani, O. P., Continuous and discontinuous solutions for optimum thrust nozzles of given length. J. Optimiz. Theory and Appl., Vol. 12, № 6, 1973.
3. Shipilin, A. V., Optimum shapes of bodies with attached shock waves. Izv. Akad. Nauk SSSR, MZhG, № 4, 1966.
4. Kraiko, A. N., Variational problems in gasdynamics of equilibrium and nonequilibrium flows. PMM Vol. 28, № 2, 1964.
5. Lur'e, K. A., Optimum Control in Problems of Mathematical Physics. "Nauka", Moscow, 1975.
6. Borisov, V. M. and Shipilin, A. V., On maximum thrust nozzles with arbitrary isoperimetric conditions. PMM Vol. 28, № 1, 1964.
7. Chernyi, G. G., Flows of Gas at High Supersonic Velocities. Fizmatgiz, Moscow, 1959.
8. Kraiko, A. N. and Tilliaeva, N. I., On constructing the contour of minimum wave drag in an inhomogeneous supersonic flow. PMM Vol. 37, № 3, 1973.
9. Kraiko, A. N. and Tilliaeva, N. I., Solution of the variational problem of constructing the contour of a compound nozzle. PMM Vol. 35, № 4, 1971.

Translated by J. J. D.

UDC 534.222.2

ON THE BEHAVIOR OF SOLUTIONS OF EQUATIONS FOR DOUBLE WAVES IN THE NEIGHBORHOOD OF THE QUIESCENT REGION

PMM Vol. 39, № 6, 1975, pp. 1043-1050

S. V. VERSHININ and A. F. SIDOROV
(Sverdlovsk)

(Received January 6, 1975)

The structure of solutions of gasdynamic equations is investigated in the case of unsteady double waves in the neighborhood of the quiescent region. A general concept of double waves is presented in the form of special series with logarithmic terms. Results of numerical computations are given.

The problem of determining the flow of plane and three-dimensional waves separated from the quiescent region by a weak discontinuity was considered in [1—3], where approximate solutions were derived for that neighborhood, and the formulation of boundary value problems required for solving the equation for the analog of the velocity potential in the hodograph plane was investigated.

The more general problem (without the assumption of the degeneration of motion) of arbitrary potential flows of polytropic gas adjacent to the quiescent region and separated by a weak discontinuity was considerd in [4—8]. Solution of that problem was obtained in the form of special series in powers of the modulus of the velocity vector r in the space of the time hodograph. The value $r = 0$ corresponds to the surface of weak discontinuity that separates the perturbed motion region from that at rest. Some applications of derived solutions to problems such as the motion of a convex piston and the propagation of weak shock waves were also investigated in those papers. Convergence in the small of obtained series was proved in [9]. However the attempts of constructing series in powers of r, which were used in [4—8] for the presentation of equations of double waves in the neighborhood of the quiescent region, proved to be unsuccessful.

Although parts of expansions in series in powers of r (accurate to within $O(r^2)$), were constructed in [1—3], it was found that the coefficient at r^3 in equations for double waves cannot be determined owing to the insolvability of its equation. This is related to the fact that the surface $r = 0$ in the case of equations for double waves is simultaneously a line of parabolic degeneration and a characteristic.

The object of the present note is the formulation of solutions of equations for plane unsteady double waves in the neighborhood of the quiescent region. Parts of the derived series, which generally are nonanalytic functions of r, can be used for defining flows at small r in particular those downstream of two-dimensional normal detonation waves [10] or in problems of angular pistons [11]. The method used for the derivation of series can be also applied in investigations of three-dimensional self-similar flows with variables x_1/x_3 and x_2/x_3 (steady flows) or x_1/t, x_2/t and x_3/t (unsteady flows). However it was not possible to obtain in such cases regular series in powers of r.

1. In polar coordinates r, φ ($u_1 = r\cos\varphi$, $u_2 = r\sin\varphi$) the system of equations defining unstable double waves in a polytropic gas is of the form

$$\frac{\gamma-1}{2}\,\theta\left[\theta_{rr}\left(1-\frac{\theta_\varphi^2}{r^2}\right)+\frac{1-\theta_r^2}{r^2}\,\theta_{\varphi\varphi}+2\,\frac{\theta_r\theta_\varphi}{r^2}\,\theta_{r\varphi}+\right.$$
$$\left.\frac{\theta_r}{r}(1-\theta_r^2)-2\,\frac{\theta_r\theta_\varphi^2}{r^3}\right]+\frac{\gamma-3}{2}\left(\theta_r^2+\frac{\theta_\varphi^2}{r^2}\right)+2=0 \tag{1.1}$$

$$\Phi_{rr}\left(1-\frac{\theta_\varphi^2}{r^2}\right)+\frac{1-\theta_r^2}{r^2}\,\Phi_{\varphi\varphi}+2\,\frac{\theta_r\theta_\varphi}{r^2}\,\Phi_{r\varphi}+$$
$$\frac{\Phi_r}{r}(1-\theta_r^2)-2\,\frac{\theta_r\theta_\varphi}{r^3}\,\Phi_\varphi=0,\quad \theta(r,\varphi)=\frac{2}{\gamma-1}\,c(r,\varphi) \tag{1.2}$$

where $c(r, \varphi)$ is the speed of sound, γ is the adiabatic exponent, $\Phi(r, \varphi)$ is the analog of the velocity potential in the hodograph plane, u_1 and u_2 are components of the velocity vector, and subscripts r and φ denote differentiation with respect to r and φ, respectively. Restitution of the flow in physical coordinates x_1, x_2, t is carried out by formulas

$$x_1 = \left[r\cos\varphi + \frac{\gamma-1}{2}\theta\left(\theta_r\cos\varphi - \theta_\varphi\frac{\sin\varphi}{r}\right)\right]t + \Phi_r\cos\varphi - \Phi_\varphi\frac{\sin\psi}{r} \quad (1.3)$$

(a similar formula is obtained for x_2 by the substitution in (1.3) of $\varphi - \pi/2$ for φ).

By setting the speed of sound $c = 1$ in the quiescent gas, we obtain for Eq. (1.1) at $r = 0$ the following Cauchy conditions [2]:

$$\theta(0,\varphi) = \frac{2}{\gamma-1}, \quad \theta_\varphi(0,\varphi) = 0, \quad |\theta_r(0,\varphi)| = 1 \quad (1.4)$$

The scheme of further exposition is as follows. First, we consider the case of the symmetric hodograph $\theta = \theta(r)$. We determine function $\theta = \theta(r)$ in the neighborhood of point $r = 0$ by eliminating successive terms of the expansion of $\theta(r)$. Elimination of these terms is effected by successive linearization of related equations and estimating the order of eliminated terms.

Later, in Sect. 2 in the analysis of the case of asymmetric hodograph we apply the method of undetermined coefficients, taking as the basis the form of solution for the symmetric hodograph.

Thus for $\theta = \theta(r)$ we have the Cauchy problem

$$\frac{\gamma-1}{2}\theta\left[\theta_{rr} + \frac{\theta_r(1-\theta_r^2)}{r}\right] + \frac{\gamma-3}{2}\theta_r^2 + 2 = 0 \quad (1.5)$$

$$\theta(0) = \frac{2}{\gamma-1}, \quad |\theta_r(0)| = 1 \quad (1.6)$$

Without loss of generality, we consider the case of $\theta_r(0) = 1$. First, we investigate the kind of the singular point of Eq. (1.5) with initial conditions (1.6). Since (1.5) is a generalized homogeneous equation, hence after the substitution $r = e^t$, $\theta = u(t)e^t$, $du/dt = p$, $p = y - u$, $u = 1/x$ and $y = z + 1$ it reduces to the form

$$\frac{dz}{dx} = \left[-\frac{\gamma+1}{2}z(z+1)(z+2) + \frac{\gamma-3}{2}(z+1)^2x + 2x\right]\times \quad (1.7)$$
$$\left\{\frac{\gamma-1}{2}x[x(z+1)-1]\right\}^{-1}$$

with singular point (0, 0). Retaining in (1.7) the linear terms, we obtain the equation

$$\frac{dw}{dx} = \left[\frac{\gamma+1}{2}x - (\gamma-1)w\right]\left(-\frac{\gamma-1}{2}x\right)^{-1} \quad (1.8)$$

The integral curves of Eq. (1.8) are of the form $w = Cx^2 + (\gamma+1)(\gamma-1)^{-1}x$ and the singular point is a node. The two exceptional directions in the wx-plane are determined by the relationships $\operatorname{tg}\varphi_1 = \infty$, $\operatorname{tg}\varphi_2 = (\gamma+1)(\gamma-1)^{-1}$.

It follows from the qualitative theory of differential equations [12] that segment $S(\delta) : \{|\varphi - \varphi_2| \leqslant \delta\}$ is parabolic (all trajectories observed in a reasonably small neighborhood of the singular point enter the latter with one of their ends and reach the region boundary with the other). From Lon's theorem we obtain the uniqueness of the discernibility problem and, consequently, the kind of singular point is determined by the linear part of expansions; hence the considered point is in fact a node.

The solution of Eq. (1.5) with initial conditions (1.6) is derived as a formal series (ε_0 is taken from [2])

$$\theta(r) \sim \sum_{i=0}^{\infty} \varepsilon_i(r), \quad \varepsilon_i(r) \sim o(r^{i+1}), \quad \varepsilon_0 = \frac{2}{\gamma-1} + r + \frac{\gamma+1}{4} r^2 \tag{1.9}$$

Let us describe the algorithm of successive steps in the determination of functions $\varepsilon_i(r)$. First, we represent $\theta(r)$ in the form

$$\theta(r) = \theta_1(r) + o(\varepsilon_1) = \varepsilon_0(r) + \varepsilon_1(r) + o(\varepsilon_1) \tag{1.10}$$

$$\varepsilon_1(r) \sim o(r^2) \quad (\varepsilon_1' \sim o(r),\ \varepsilon_1'' \sim o(1)) \tag{1.11}$$

The substitution of (1.10) into (1.5) yields for $\varepsilon_1(r)$ the nonlinear equation

$$\varepsilon_1'' + \frac{1}{r}[\varepsilon_0' + \varepsilon_1' + o(\varepsilon_1')][1 - (\varepsilon_0' + \varepsilon_1' + o(\varepsilon_1'))^2] + \varepsilon_0'' + o(\varepsilon_1'') + F_1(r) = 0 \tag{1.12}$$

$$F_1(r) = \frac{{}^1/_2(\gamma-3)(\varepsilon_0' + \varepsilon_1' + o(\varepsilon_1'))^2 + 2}{{}^1/_2(\gamma-1)(\varepsilon_0 + \varepsilon_1 + o(\varepsilon_1))} \approx \frac{\gamma+1}{2}\left(1 + \frac{\gamma-5}{2} r + o(r)\right)$$

Retaining in Eq. (1.12) terms of order ε_1'' and discarding those of order $o(\varepsilon_1'')$, we obtain for $\varepsilon_1(r)$ the linear equation

$$\varepsilon_1'' - \frac{2}{r}\varepsilon_1' = \frac{(\gamma+1)(\gamma+4)}{2} r$$

which after integration with allowance for (1.11) yields

$$\varepsilon_1(r) = \frac{(\gamma+1)(\gamma+4)}{6} r^3 \ln r + \left[\frac{1}{3}C - \frac{(\gamma+1)(\gamma+4)}{18}\right] r^3$$

where C is an arbitrary constant.

The second step of this method yields

$$\varepsilon_2(r) = \frac{(\gamma+1)(\gamma+3)(\gamma+4)}{4} r^4 \ln r + \Big\{-\frac{5(\gamma+1)(\gamma+3)(\gamma+4)}{16} + \frac{1}{4}\Big[\frac{\gamma+1}{48}(25\gamma^2 + 100\gamma + 243) + 6(\gamma+3) \times \left(\frac{1}{3}C - \frac{(\gamma+1)(\gamma+4)}{18}\right)\Big]\Big\} r^4$$

In the n-th step the sought solution is of the form

$$\theta(r) = \theta_n + o(\varepsilon_n) = \theta_{n-1} + \varepsilon_n + o(\varepsilon_n) = \sum_{i=0}^{n-1} \varepsilon_i + \varepsilon_n + o(\varepsilon_n) \tag{1.13}$$

where $\varepsilon_n(r)$ must satisfy condition

$$\varepsilon_n(r) \sim o(r^{n+1}), \quad (\varepsilon_n' \sim o(r^n), \quad \varepsilon_n'' \sim o(r^{n-1})) \tag{1.14}$$

Substituting (1.13) into (1.5) we obtain

$$\frac{\gamma-1}{2}(\theta_{n-1} + \varepsilon_n + o(\varepsilon_n))\Big\{\varepsilon_n'' + \frac{1}{r}(\theta_{n-1}' + \varepsilon_n' + o(\varepsilon_n'))[1 - (\theta_{n-1}' + \varepsilon_n' + o(\varepsilon_n'))^2] + \theta_{n-1}'' + o(\varepsilon_n'')\Big\} + \frac{\gamma-3}{2}[\theta_{n-1}' + \varepsilon_n' + o(\varepsilon_n')]^2 + 2 = 0 \tag{1.15}$$

Estimates of all terms of Eq. (1.15) yield the equation

$$\varepsilon_n'' - \frac{2}{r}\varepsilon_n' = -\frac{\gamma-1}{2}\theta_{n-1}\left[\frac{1}{r}\theta_{n-1}'(1-\theta_{n-1}'^2)+\theta_{n-1}''\right] - \frac{\gamma-3}{2}\theta_{n-1}'^2 - 2 + o(\varepsilon_n''). \tag{1.16}$$

By definition the terms of Eq. (1.16) of order lower than r^{n-1} cancel out. The rejection of terms of order $o(\varepsilon_n'')$ yields the n-th approximation equation

$$\varepsilon_n'' - \frac{2}{r}\varepsilon_n' = r^n \sum_{i=0}^{k_n} b_i^{(n)}(\gamma, C)\ln^i r \tag{1.17}$$

where $k_n = m$ for $n = 2m-1$ or $n = 2m$; $b_1^{(1)} \equiv 0$ and $b_i^{(n)}$ are constants that depend on γ and C whose form can be determined with the use of recurrent formulas. In accordance with condition (1.14) the constants of integration are assumed to be identically zero, hence the integration of (1.17) yields

$$\varepsilon_n(r) = r^{n+2}\sum_{i=0}^{k_n} d_i^{(n)}(\gamma, C)\ln^i r$$

(the structure of $d_i^{(n)}(\gamma, C)$ is similar to that of $b_i^{(n)}(\gamma, C)$).

Thus the formal series

$$\theta(r) \sim \frac{2}{\gamma-1} + \sum_{\substack{n=0\\ m=1}}^{\infty} a_{nm}(\gamma, C)\, r^{m+2n}\ln^n r \tag{1.18}$$

$$(a_{01} = 1,\ a_{02} = {}^1/_4(\gamma+1),\ a_{11} = {}^1/_6(\gamma+1)(\gamma+4),\ a_{03} = {}^1/_3 C - {}^1/_{18}(\gamma+1)(\gamma+4))$$

can be considered as corresponding to the solution $\theta(r)$ of Eq. (1.5) with initial conditions (1.6).

Note. It was shown in [13] that in the case of cylindrical symmetry with the self-similar variable $\sqrt{x_1^2+x_2^2}/t$ in the region of weak discontinuity $-\xi - C_0 = 2\alpha_0 v + \text{const}\ v^2$ ($v(r,t) = v_r$, $\xi = r/t$, C_0 is the speed of sound at $v = 0$, and α_0 is a positive constant), which is the zero term of the expansion of $\theta(r)$. The descibed recurrent procedure makes it possible to construct the complete expansion.

2. In the case of an asymmetric hodograph ($\theta = \theta(r, \varphi)$) the solution of Eq. (1.1) with initial conditions (1.4) is sought by analogy with (1.18) in the form

$$\theta(r, \varphi) = \frac{2}{\gamma-1} + \sum_{\substack{n=0\\ m=1}}^{\infty} a_{nm}(\varphi)\, r^{m+2n}\ln^n r \tag{2.1}$$

To determine the unknown coefficients $a_{nm}(\varphi)$ we substitute (2.1) into (1.1) and equate to zero the coefficients at terms of the form $r^s \ln^q r$.

Let us describe the procedure of successive determination of $a_{nm}(\varphi)$. For this we write the explicit formula for the coefficients T_{sq} at $r^s\ln^q r$ in Eq. (1.1) into which we substitute (2.1). We obtain

$$T_{sq} = -\frac{\gamma-1}{2}\Big\{\sum_{\substack{M_4=s-2q\\N_4=q}} p_2[\Pi_1-(2m_4-m_2+4n_4-2n_2)\Pi_2+ \tag{2.2}$$

$$p_3\Pi_3]+\sum_{\substack{M_4=s-2q-2\\N_4=q+1}}\Big(\sum_{\substack{i,j,k=2\\i\neq j\neq k}}^{4} n_ip_jp_k\Big)\Pi_1+[n_2(p_2{}^2-1)-2(\alpha_{24}+$$

$$4n_2m_4+n_4)]\Pi_2+[\alpha_{24}+4\beta_{23}+(n_2+n_3)]\Pi_3+$$

$$\sum_{\substack{M_4=s-2q-4\\N_4=q+2}}\Big(\sum_{\substack{i,j,k=2\\i\neq j\neq k}}^{4}\beta_{ij}p_k\Big)\Pi_1-n_2[2n_4-(p_2-1)(n_2-1)]\Pi_2+$$

$$\beta_{23}\Pi_3+\sum_{\substack{M_4=s-2q-6\\N_4=q+3}}\delta_{234}\Pi_1\Big\}-\sum_{\substack{M_3=s-2q+1\\N_3=q}} p_1[p_2p_3\Pi_4+(2(n_1-2n_3)+$$

$$(m_1-2m_3))\Pi_5+p_2\Pi_6]-\sum_{\substack{M_3=s-2q-1\\N_3=q+1}}\Big(\sum_{\substack{i,j,k=1\\i\neq j\neq k}}^{3} n_ip_jp_k\Big)\Pi_4+$$

$$[n_1(p_1{}^2-1)-2(\alpha_{13}+4\beta_{13}+n_3)]\Pi_5+(\alpha_{12}+4\beta_{12}+N_2)\Pi_6-$$

$$\sum_{\substack{M_3=s-2q-3\\N_3=q+2}}\Big(\sum_{\substack{i,j,k=1\\i\neq j\neq k}}^{3}\beta_{ij}p_k\Big)\Pi_4+n_1[-2n_3+(p_1-1)(n_1-1)]\Pi_5+$$

$$\beta_{12}\Pi_6-\sum_{\substack{M_3=s-2q-5\\N_3=q+3}}\delta_{123}\Pi_4+\sum_{\substack{M_2=s-2q\\N_2=q}}\Big[\frac{\gamma-1}{2}p_1(p_1-1)+$$

$$\frac{\gamma-1}{2}p_2+\frac{\gamma-3}{2}p_1p_2\Big]\Pi_7+\frac{\gamma-3}{2}\Pi_8+\frac{\gamma-1}{2}\Pi_9+$$

$$\sum_{\substack{M_2=s-2q-2\\N_2=q+1}}\Big[(\gamma-1)n_1p_1+\frac{\gamma-3}{2}(\alpha_{12}+4\beta_{12}+N_2)\Big]\Pi_7+$$

$$\sum_{\substack{M_2=s-2q-4\\N_2=q+2}}\Big[\frac{\gamma-1}{2}n_1(n_1-1)+\frac{\gamma-3}{2}n_1n_2\Big]\Pi_7+(s+2)^2a_{qs-2q+2}+$$

$$a''_{qs-2q+2}+2(q+1)(s+2)a_{q+1s-2q}+(q+1)(q+2)a_{q+2s-2q-2}$$

where

$$\Pi_1=a_{n_1m_1+1}a_{n_2m_2+1}a_{n_3m_3+1}a_{n_4m_4+1}$$

$$\Pi_2=a_{n_1m_1+1}a_{n_2m_2+1}a'_{n_3m_3+1}a'_{n_4m_4+1},$$

$$\Pi_3=a_{n_1m_1+1}a_{n_2m_2+1}a_{n_3m_3+1}a''_{n_4m_4+1}$$

$$\Pi_4=a_{n_1m_1+1}a_{n_2m_2+1}a_{n_3m_3+1},\qquad \Pi_5=a_{n_1m_1+1}a'_{n_2m_2+1}a'_{n_3m_3+1}$$

$$\Pi_6=a_{n_1m_1+1}a_{n_2m_2+1}a''_{n_3m_3+1},\qquad \Pi_7=a_{n_1m_1+1}a_{n_2m_2+1}$$

$$\Pi_8=a'_{n_1m_1+1}a'_{n_2m_2+1},\qquad \Pi_9=a_{n_1m_1+1}a''_{n_2m_2+1}$$

$$\sum_{i=1}^{k}m_i=M_k,\qquad \sum_{i=1}^{k}n_i=N_k,\qquad k=2,3,4$$

$$m_i+2n_i+1=p_i,\qquad n_in_k=\beta_{ik},\qquad n_im_k+n_km_i=\alpha_{ik},$$

$$n_in_kn_j=\delta_{ikj},\qquad k=1,2,3,4$$

For the first coefficients from (2.2) we obtain

$$T_{-10} = (a_{01}'' + a_{01})(1 - a_{01}^2) = 0 \tag{2.3}$$

By virtue of initial conditions (1.4) $a_{01}^2 = 1$ and the equality (2.3) is automatically satisfied. From conditions $T_{00} = 0$ and $T_{11} = 0$ we similarly obtain $a_{02} = {}^1/_4\,(\gamma + 1)$ and $a_{11} = l\,(\varphi)$, where $l\,(\varphi)$ is an arbitrary function. We define coefficients a_{nm} in the form of matrix

$$A_\infty = \left\| \begin{array}{llll} a_{00} & & & \\ a_{01} & & & \\ a_{02} & & & \\ a_{03} & a_{11} & & \\ a_{04} & a_{12} & & \\ \cdots & \cdots & \cdots & \cdots \\ a_{0\,2k-1} & a_{1\,2k-3} & \cdots & a_{k-1\,1} \\ a_{0\,2k} & a_{1\,2k-2} & \cdots & a_{k-1\,2} \\ \cdots & \cdots & \cdots & \cdots \end{array} \right\|$$

where empty spaces represent zero elements.

Elements of matrix A_∞ are successively determined from top to bottom and from left to right. We will show that by equating to zero all T_{sq}, it is possible to determine consecutively all a_{nm}. The position of coefficients $a_{n_* m_*}$ in (2.2), for which $m_* = \max m$ (for fixed s and q) and $n_* = \min n$ (for fixed s, q and m_*), is determined by the quantity $M_k + 2N_k + 1$. For it, $M_k + 2N_k + 1 \leqslant s + 2$. The equality corresponds exactly to $a_{n_* m_*}$.

Having determined $2k$ rows in A_∞, we obtain the $(2k+1)$-st and the $(2k+2)$-nd rows with the use of the recurrent formulas

$$\begin{gathered} T_{2k-1q} = (2k+1)(2k-2)\,a_{q2k-2q+1} - f_{q2k-2q+1}(a_{00}, \ldots, a_{q_*2k-2q_*+1}) = 0 \\ T_{2kq} = (2k-1)(2k+2)\,a_{q2k-2q+2} - f_{q2k-2q+2}(a_{00}, \ldots, a_{q_*2k-2q_*+2}) = 0 \\ (k \geqslant q_* > q \geqslant 0) \end{gathered} \tag{2.4}$$

Hence for $k \neq 1$ the use of (2.4) makes possible the recurrent determination of a_{nm} ($n = k, k-1, \ldots, 0$; $m = 1, 3, \ldots, 2k+1$ and $m = 2, 4, \ldots, 2k+2$).

The case of $k = 1$ relates to constant arbitrariness of the determination of f_{ij} beginning with a_{03} for the symmetric hodograph ($\theta = \theta\,(r)$) and to a single-functional arbitrariness commencing with a_{11} in the case of the asymmetric hodograph. In this manner the concept of $\bar{\theta}\,(r, \varphi)$ is established.

The described algorithm makes it possible to determine with the use of expansions (1.18) and (2.1) the form of solution for the velocity potential (1.2). For $\Phi\,(r, \varphi)$ of the form

$$\Phi(r, \varphi) \sim b_{01} r + \sum_{\substack{n=0 \\ m=2}}^{\infty} b_{nm}(\varphi)\, r^{m+2n} \ln^n r$$

the coefficients b_{nm} are determined by the related recurrent formulas

$$(2k+1)\,(2k+2)\,b_{q2k-2q+2} = \psi_{q2k-2q+2}\,(b_{01}, \ldots, b_{q_*2k-2q_*+2}, a_{00}, \ldots, a_{q_*2k-2q_*+1})$$

$$(2k+2)\ (2k+3)\ b_{q2k-2q+3} = \psi_{q2k-2q+3}\,(b_{01},\ \ldots,\ b_{q_*2k-2q_*+3},$$
$$a_{00},\ \ldots,\ a_{q_*2k-2q_*+2})$$
$$(k \geqslant q_* > q \geqslant 0,\ \ k = 0,\ 1,\ 2,\ \ldots),$$

where $b_{01} = l_1\,(\varphi)$ is an arbitrary function.

The formulation of solutions of Eqs. (1.1) and (1.2) for double waves is thus completed. The arbitrariness of this formulation is due to two functions $l\,(\varphi)$ and $l_1\,(\varphi)$ of one argument. With the use of $l_1\,(\varphi)$ it is possible to specify the shape of a weak discontinuity for any instant of time $t = t_0$, while $l\,(\varphi)$ is determined by the specific physical conditions of the problem. Convergence of the derived series has not been so far established.

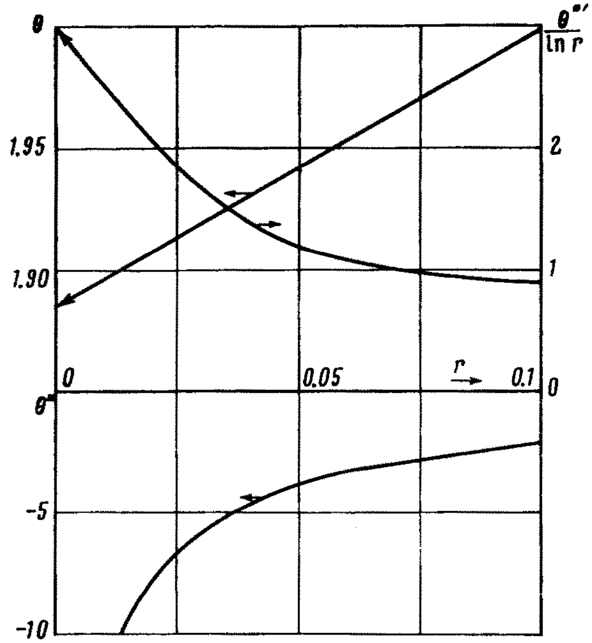

Fig. 1

3. Numerical computations were carried out for the ordinary differential equation (1.5) with boundary conditions [10]

$$r = \frac{D}{\gamma+1}, \quad \theta\,(r) = \frac{2\gamma D}{\gamma^2-1}, \quad \theta_r\,(r) = 1$$

(D is the velocity of the detonation wave front) up to $r = 0$.

These boundary conditions correspond to the Chapman-Jouguet conditions at the front of a cylindrical detonation wave initiated at instant $t = 0$ along an infinite axis. Function $\theta\,(r)$ corresponds to the distribution of the speed of sound behind the wave in terms of the velocity of products of explosion. The value $\theta\,(0)$ corresponds to the propagation velocity of a weak discontinuity that separates the region of motion from that of quiescent products of explosion.

Curves of θ, θ'', $\theta''/\ln r$ ($\gamma = 3$, $D = 4$) are shown in Fig. 1. It will be seen from these that in some region $r = 0$ (C_1, C_2 and K are constants) the values $\theta \sim 1 + r$, $\theta''' \sim C_1 + C_2 \ln r$ and $|\theta''' / \ln r| < K$ are valid for θ, θ''' and $\theta'''/\ln r$. Thus numerical computations confirm the presence of the term $r^3 \ln r$ in the expansion of function $\theta(r)$.

REFERENCES

1. Sidorov, A. F., On the flow of gas in the neighborhood of a weak discontinuity. Dokl. Akad. Nauk SSSR, Vol. 167, № 1, 1966.
2. Sidorov, A. F., On unsteady gas flows adjacent to the region at rest. PMM Vol. 30, № 1, 1966.
3. Sidorov, A. F., Some three-dimensional gas flows adjacent to region at rest. PMM Vol. 32, № 3, 1968.
4. Sidorov, A. F., Disruption of potential gas flows adjacent to the region at rest. PMM Vol. 35, № 3, 1971.
5. Sidorov, A. F., On the solution of certain boundary value problems in the theory of potential flows of gas and the propagation of weak shock waves. Dokl. Akad. Nauk SSSR, Vol. 204, № 4, 1972.
6. Sidorov, A. F., Method of solution of certain boundary value problems for nonlinear hyperbolic equations and propagation of weak shock waves. PMM Vol. 36, № 3, 1972.
7. Zubov, E. N. and Sidorov, A. F., On the solution of a certain boundary value problem for unsteady three-dimensional gas flow and the propagation of weak spherical shock waves. Collection: Numerical Methods in Mechanics of Continuous Medium, Vol. 3, № 3, VTs SO Akad. Nauk SSSR, Novosibirsk, 1972.
8. Gavrilushkin, I. B. and Sidorov, A. F., On a class of solutions of the nonlinear equation for the velocity potential. PMM Vol. 38, № 2, 1974.
9. Bautin, S. P., Analytic solution of the problem of piston motion. Collection: Numerical Methods in Mechanics of Continuous Medium, Vol. 4, № 1, VTs SO Akad. Nauk SSSR, Novosibirsk, 1973.
10. Sidorov, A. F., On certain exact solutions of unsteady two-dimensional gasdynamics. PMM Vol. 26, № 2, 1962.
11. Ermolin, E. V., Rubina, L. I. and Sidorov, A. F., On the problem of two pistons. PMM Vol. 32, № 5, 1968.
12. Nemytskii, V. V. and Stepanov, V. V., The Qualitative Theory of Differential Equations. Gostekhizdat, Moscow-Leningrad, 1947.
13. Landau, L. D. and Lifshits, E. M., Mechanics of Continuous Media. Gostekhizdat, Moscow, 1954.

Translated by J. J. D.